Effect of Chemical Reaction on MHD Free Convection Flow past a Vertical Plate with Variable Temperature and Variable Concentration

Bhaben Ch. Neog, Rudra Kr. Das, Rudra Kt. Deka

Abstract— Laplace transform method is used to study the effect of magnetic field on transient free convection flow of an electrically conducting fluid over an impulsively started vertical plate with variable temperature and variable concentration. Solutions obtained are presented graphically for different values of physical parameters. It is observed that chemical reaction parameter and magnetic parameter influence the velocity, temperature and concentration profiles significantly.

KeyWords: Free Convection, Vertical Plate, MHD, Variable Temperature, Variable Concentration, Chemical Reaction
AMS 2000 subject classification: 76R10, 76W05, 80A20, 80A32

1 Introduction

Mass transfer with chemical reaction is one of the most commonly encountered circumstances in chemical industry as well as in physical and biological sciences. In some other areas such as food processing industry, paper processing technology, the evaporation or condensation process, solvent extraction, humidification, sublimation, oxygenation of blood, food and drug assimilation, respiration mechanism, etc. chemical reaction takes place. When mass transfer takes place in a fluid at rest, the mass is transferred purely by molecular diffusion resulting from concentration gradients. For low concentration of the mass in the fluid and low mass transfer rates, the convective heat and mass transfer processes are similar in nature. A number of investigations have already been carried out with combined heat and mass transfer under the assumption of different physical situations.

Further, magneto-convection plays an important role in industrial and engineering applications such as liquid metal cooling in nuclear reactors, magnetic control of molten iron flow in steel industry etc. Hence combined study will surely enhance the already developed areas further for more complex studies.

In many cases in the process of free convection, chemical reaction also takes place due to the presence of foreign masses (as impurities) in fluid. It is found that in many chemical engineering processes, chemical reaction takes place between foreign masses (present in the form of ingredients) and the fluid. This type of chemical reaction may change the temperature and the heat content of the fluid and may affect the free convection process. However if the presence of such foreign mass is very low then we can assume the first order chemical reaction so that heat generation due to chemical reaction can be considered to be very negligible. Here only first order chemical reaction is considered. A reaction is said to be of the first order if the rate of reaction is directly proportional to the concentration.

The effects of mass transfer on free convection flow past a semi-infinite vertical isothermal plate was first studied by Gebhart and Pera [8]. Effects of transverse magnetic field on free convection flow past an impulsively started vertical plate was first studied by Soundalgekar *et. al.*[14] and the effects of mass transfer on the flow past an impulsively started infinite vertical plate with variable temperature was studied by Soundalgekar *et. al.* [15]. Das *et. al.* [2], [3], [4] studied the effects of mass transfer on free convection flow past an impulsively started infinite vertical plate with chemical reaction, constant heat flux and with periodic temperature variation. Muthucumaraswamy *et. al.* [10] considered the effects of mass transfer on impulsively started infinite vertical plate with variable temperature and uniform mass flux. All of them considered the fact that free convection current caused by temperature differences is also caused by the differences in concentration or material constitution as suggested by Gebhart [7].

Muthucumaraswamy and Meenakshisundaram [11] studied the chemical reaction effects on vertical oscillating plate with variable temperature and chemical reaction. Deka and Neog [5] considered the combined effects of thermal radiation and chemical reaction on free convection flow past a vertical plate in porous medium. Deka and Neog [6] also studied the effect of MHD free convection flow past an oscillating plate with thermal radiation and variable mass diffusion.

Very recently, Neog [12] studied the unsteady MHD flow past a vertical oscillating plate with variable temperature and chemical reaction and Muthucumaraswamy *et. al.* [11] studied the effect of thermal radiation MHD flow past a vertical oscillating plate with chemical reaction of first order.

Although many authors studied mass transfer with or without chemical reaction in flow past oscillating vertical plate by considering different surface conditions but the study on the effects of magnetic field on free convection heat and mass transfer in the presence of transverse magnetic field and chemical reaction with variable temperature has not been found in

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literature and hence the motivation to undertake this study. It is therefore proposed to study the effects of chemical reaction on hydromagnetic flow past a vertical plate with variable temperature under the assumption of first order chemical reaction

2 MATHEMATICAL ANALYSIS

An unsteady natural convection flow of a viscous incompressible electrically conducting fluid past an infinite vertical plate is considered here. To visualize the flow pattern a Cartesian co-ordinate system is considered where x'-axis is taken along the infinite vertical plate, the y'-axis is normal to the plate and fluid fills the region $y' \geq 0$. Initially, the fluid and the plate are kept at the same constant temperature T'_∞ and species concentration C'_∞. At time $t' > 0$, the plate is given a uniform motion in its own plane with a velocity U_0. And at the same time the plate temperature and species concentration raised linearly with time and a magnetic field of uniform strength B_0 is applied normal to the plate. It is assumed that the magnetic Reynolds number is very small and the induced magnetic field is negligible in comparison to the transverse magnetic field. It is also assumed that the effect of viscous dissipation is negligible in the energy equation and the level of species concentration is very low so the Soret and Dufour effects are negligible.

As the plate is infinite in extent so the derivatives of all the flow variables with respect to x' vanish and they can be assumed to be functions of y' and t' only. Thus the motion is one dimensional with only non-zero vertical velocity component u', varying with y' and t' only. Due to one dimensional nature, the equation of continuity is trivially satisfied.

Under the above assumptions and following Boussinesq approximation, the unsteady flow field is governed by the following set of equations:

Momentum equation:

$$\frac{\partial u'}{\partial t'} = g\beta(T'-T'_\infty) + g\beta^*(C'-C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}u' \tag{1}$$

Energy equation:

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} \tag{2}$$

Diffusion equation:

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_1 C' \tag{3}$$

Along with the following initial and boundary conditions:

$$\begin{array}{l} u'=0, \quad T'=T'_\infty, \quad C'=C'_\infty \quad \text{for all } y' \text{ and } t'\leq 0 \\ \left.\begin{array}{l} u'=U_0, \quad T'=T'_\infty+(T'_w-T'_\infty)At', \quad C'=C'_\infty+(C'_w-C'_\infty)At' \text{ at } y'=0 \\ u'\to 0, \quad T'\to T'_\infty, \quad C'\to C'_\infty \quad \text{as } y'\to\infty \end{array}\right\}, t>0 \end{array} \tag{4}$$

where u', v' are the velocity components in x', y' directions respectively, t' the time, A is a constant, B_0 Magnetic field strength along y' axis, C' dimensional concentration, C_p the specific heat at constant pressure, D mass diffusivity, g the acceleration due to gravity, k thermal conductivity, K_1 chemical reaction parameter, T' dimensional temperature, u dimensionless velocity, α thermal diffusivity , β and β^* the thermal and concentration expansion coefficient, θ dimensionless temperature, ϕ dimensionless concentration, ρ the fluid density, ν kinematic viscosity, σ electrical conductivity, μ coefficient of viscosity.

Now to reduce the above equations in non-dimensional form we introduce the following non-dimensional quantities.

$$\left.\begin{array}{l} u=\dfrac{u'}{U_0},\ t=\dfrac{t'U_0^2}{\nu}, y=\dfrac{y'U_0}{\nu}, \theta=\dfrac{T'-T'_\infty}{T'_w-T'_\infty}, \\ \phi=\dfrac{C'-C'_\infty}{C'_w-C'_\infty}, \quad Gr=\dfrac{g\beta\nu(T'_w-T'_\infty)}{U_0^3}, \\ Gm=\dfrac{g\beta^*\nu(C'_w-C'_\infty)}{U_0^3},\ \Pr=\dfrac{\mu C_p}{\kappa}, \\ A=\dfrac{u_0^2}{\nu}, \quad R=\dfrac{\nu K_1}{U_0^2}, \quad Sc=\dfrac{\nu}{D}, \quad M=\dfrac{\sigma B_0^2\upsilon}{\rho U_0^2} \end{array}\right\} \tag{5}$$

Thus with the help of above non dimensional quantities (5), the equations (1), (2) and (3) then reduce to the following forms.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi - Mu \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - R\phi \tag{8}$$

And the initial and boundary conditions (4) are as follows:

$$\begin{array}{l} u=0, \quad \theta=0, \quad \phi=0, \text{ for all } y \text{ and } t\leq 0 \\ \left.\begin{array}{l} u=1, \quad \theta=t, \quad \phi=t, \text{ at } y=0 \\ u\to 0, \quad \theta\to 0, \quad \phi\to 0, \text{ as } y\to\infty \end{array}\right\}, t>0 \end{array} \tag{9}$$

where Gr, Gm, Pr, M, Sc and R are Grashof number, mass Grashof number, Prandtl number, Hartmann number, Schmidt number and Chemical reaction parameter respectively.

Solutions of the equations (6), (7) and (8) subject to the initial and boundary conditions (9) are obtained with the help of Abramowtz and Stegum [1] and Hetnarski's [9] algorithm. They are obtained as follows:

$$\theta(y,t)=\left(t+\frac{y^2\,\mathrm{Pr}}{2}\right)erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}\right)-\frac{y\sqrt{t\,\mathrm{Pr}}}{\sqrt{\pi}}e^{-\frac{y^2\,\mathrm{Pr}}{4t}} \tag{10}$$

$$\begin{aligned}\phi(y,t)=&\frac{1}{2}\left\{e^{-y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Rt}\right)+e^{y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Rt}\right)\right\}\\&-\frac{y\sqrt{Sc}}{4\sqrt{R}}\left\{e^{-y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Rt}\right)-e^{y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Rt}\right)\right\}\end{aligned} \tag{11}$$

$$\begin{aligned}u(y,t)=&\frac{1}{2}\left(G_3-tG_4\right)\left\{e^{-y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)+e^{y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\right\}\\
&+\frac{y}{4\sqrt{M}}\left(G_4+\frac{G_1}{b}\right)\left\{e^{-y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)-e^{y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\right\}\\
&+\frac{G_1}{2b^2}\left[e^{bt}\left(\left\{e^{-y\sqrt{c}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{ct}\right)+e^{y\sqrt{c}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{ct}\right)\right\}\right.\right.\\
&\left.\left.-\left\{e^{-y\sqrt{b\,\mathrm{Pr}}}erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}-\sqrt{bt}\right)+e^{y\sqrt{b\,\mathrm{Pr}}}erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}+\sqrt{bt}\right)\right\}\right)+erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}\right)\right]\\
&+\frac{G_2}{2d^2}\left[e^{dt}\left(\left\{e^{-y\sqrt{f}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{ft}\right)+e^{y\sqrt{f}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{ft}\right)\right\}\right.\right.\\
&\left.-\left\{e^{-y\sqrt{hSc}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{ht}\right)+e^{y\sqrt{hSc}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{ht}\right)\right\}\right)\\
&\left.+\left\{e^{-y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Rt}\right)+e^{y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Rt}\right)\right\}\right]\\
&+\frac{G_2}{d}\left[t\left\{e^{-y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)+e^{y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\right\}\right.\\
&\left.-\frac{y}{4\sqrt{M}}\left\{e^{-y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Rt}\right)+e^{y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Rt}\right)\right\}\right]\\
&+\frac{G_1}{b}\left[\left(t+\frac{y^2\,\mathrm{Pr}}{2}\right)erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}\right)-\frac{y\sqrt{t\,\mathrm{Pr}}}{\sqrt{\pi}}e^{-\frac{y^2\,\mathrm{Pr}}{4t}}\right]\end{aligned} \tag{12}$$

Here, the following symbols are used in the above solutions:

$$\left.\begin{aligned}&b=\frac{M}{\mathrm{Pr}-1},\; c=M+b,\; d=\frac{RSc-M}{1-Sc},\; h=R+d,\\&k=M+d,\quad G_1=\frac{Gr}{\mathrm{Pr}-1}, G_2=\frac{Gm}{Sc-1},\\&G_3=1-\frac{G_2}{d^2}-\frac{G_1}{b^2},\quad G_4=\frac{G_2}{d}+\frac{G_1}{b}\end{aligned}\right\} \tag{13}$$

3 RESULTS AND DISCUSSION

To know the influence of different physical parameters viz., chemical reaction parameter, Schmidt number, thermal Grashof number, mass Grashof number, Hartmann number, Prandtl number and time on the physical flow, computations are carried out for vertical velocity, temperature and concentration and they are presented in figures.

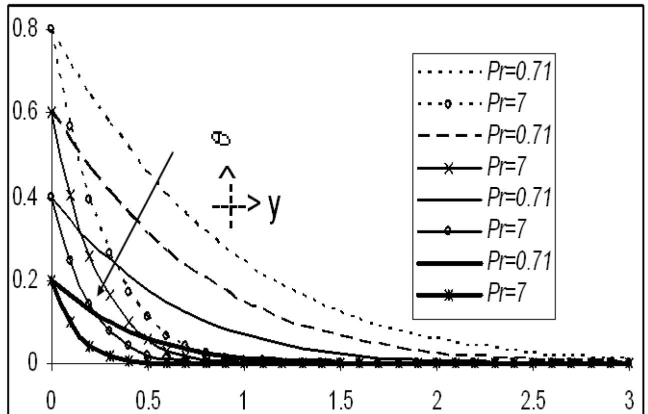

Fig-1a Temperature profile showing the effect of Pr and time

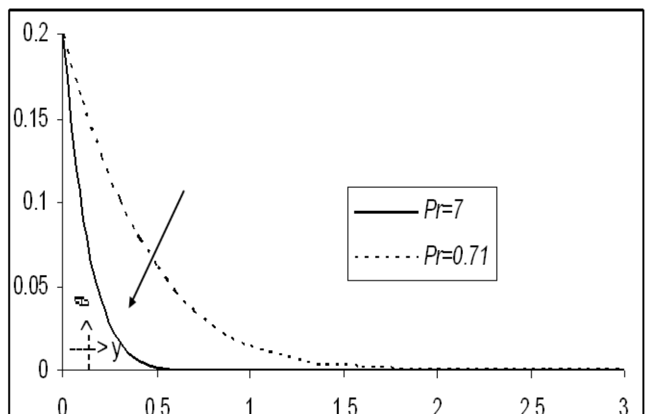

Fig-1b Temperature profile showing the effect of Pr

In figures 1a and 1b temperature profiles are plotted for Pr (7, 0.71) and time t(0.2,0.4,0.6,0.8). Since temperature is considered as time dependent, therefore this figure clearly reflects the situation. From figures it is further observed that temperature decreases as Pr increases and increases as time increases.

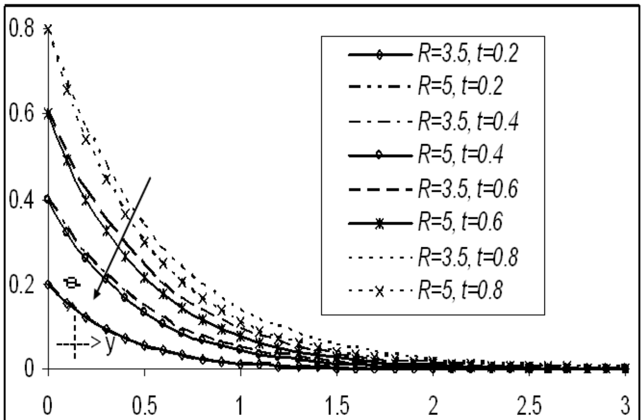

Fig-2a Concentration profile showing the effects of R & t

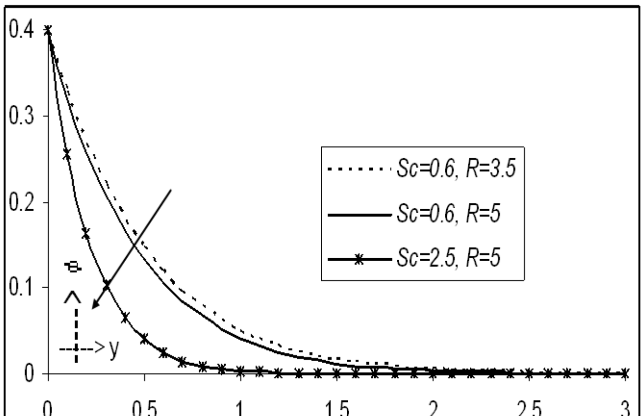

Fig-2b Concentration profile showing the effect of *Sc* & *R*

In figures 2a and 2b concentration profiles are presented for different values of *Sc* (0.6, 2.5), *R* (3.5, 5) and time (t=0.2, 0.4, 0.6, 0.8). It is observed that increase of Schmidt number and chemical reaction parameter lead to the decrease in concentration of the species. Time dependent concentration can clearly be visualized from the figures.

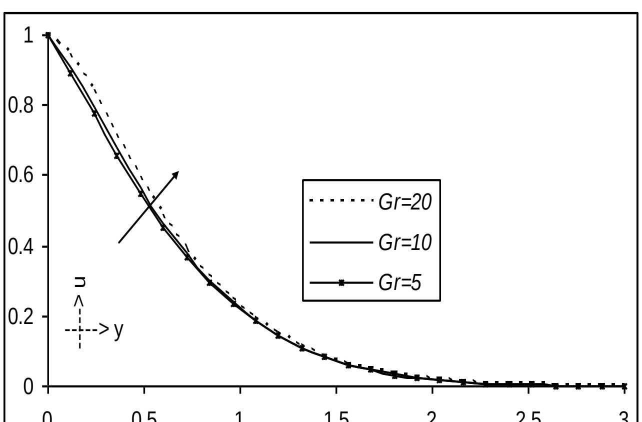

Fig-3a Velocity profile showing the efect of *Gr*

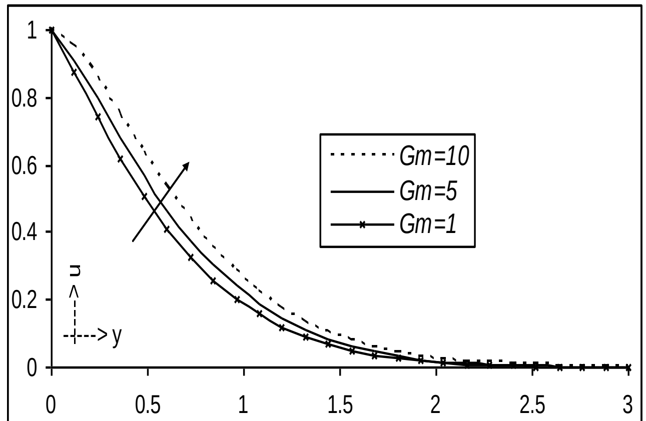

Fig-3b Velocity profile showing the effect of *Gm*

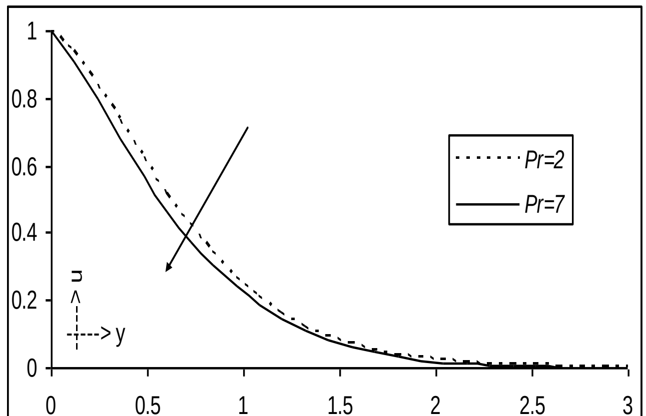

Fig-3c Velocity profile showing the effect of *Pr*

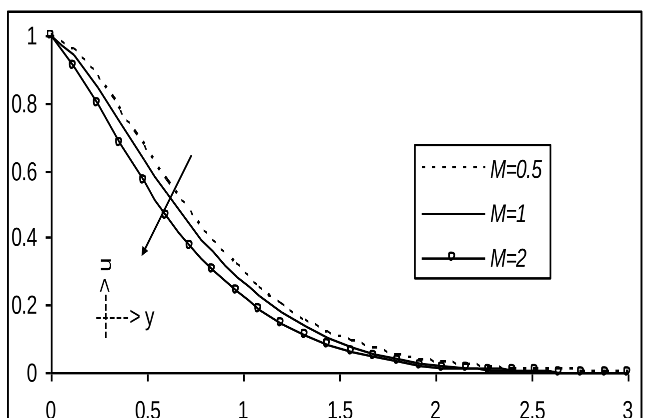

Fig-3d Velocity profile showing the effect of *M*

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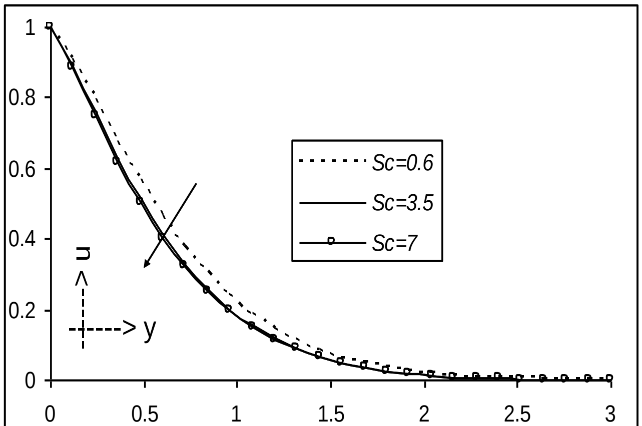

Fig-3e Effect of *Sc* on Vertical Velocity

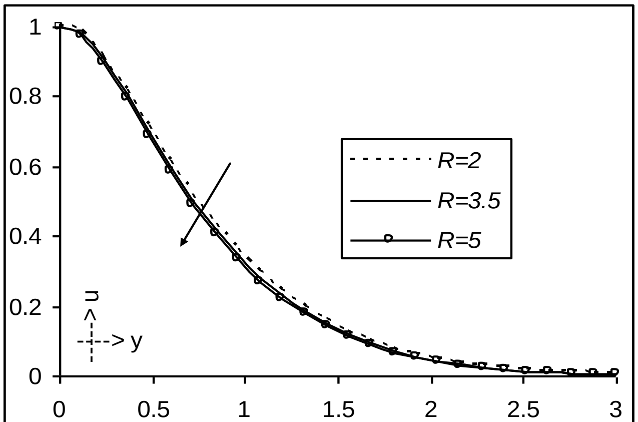

Fig-3f Effect of *R* on Vertical Velocity

In figures 3a to 3f velocity profiles are presented for different values of *Gr(5,10,20), Gm(1,5,10), Pr(0.71, 7), M(0.5,1,2), Sc(0.6,3.5,7), R(2,3.5,5)* at different times *t*. It is observed that velocity decreases with the increase of *Pr, M, Sc* and *R* but velocity increases with the increase of *Gr* and *Gm.*

4 CONCLUSIONS

An exact analysis in closed form is performed by Laplace Transform method to study the influence of chemically reacting hydromagnetic flow past a moving vertical plate with variable temperature. Some of the important conclusions of the study are as follows:

- Temperature decreases with the increase of *Pr*.
- Concentration decreases as *Sc* and *R* increase.
- Velocity increases with increasing *Gr, Gm.*
- Also increase in *M, Sc, Pr* and *R* lead to decrease in velocity.

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